

# Mechanics of binary crushable granular assembly through discrete element method



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## ABSTRACT

The mechanical response of a granular system is not only influenced by the bulk material properties but also on various factors due to its discrete nature. The factors like topology, packing fraction, friction between particles, particle size distribution etc. influence the behavior of granular systems. For a reliable design of such systems like fusion breeder units comprising of pebble beds, it is essential to understand the various factors influencing the response of the system. Mechanical response of a binary assembly consisting of crushable spherical pebbles is studied using Discrete Element Method (DEM) which is based on particle–particle interactions. The influence of above mentioned factors on the macroscopic stress–strain response is investigated using an in-house DEM code. Furthermore, the effect of these factors on the damage in the assembly is investigated. This present investigation helps in understanding the macroscopic response and damage in terms of microscopic factors paving way to develop a unified prediction tool for a binary crushable granular assembly.

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## 1. Introduction

Helium Cooled Pebble Bed (HCPB) consists of Lithium Orthosilicate (OSi) as candidate breeder material for EU, in the form of pebbles [1]. Mechanical response of such granular assembly is essential for designing the beds for a safe and sustained fusion cycle. Due to discrete nature of pebble assembly, the response is not only influenced by the bulk properties of material but also on various factors viz. Packing fraction (PF), relative radii, topology, size distribution of pebbles in the assembly. Discrete Element Method (DEM), based on particle–particle interactions, is an effective way to study the behavior of such discrete systems. DEM helps to understand microscale interactions at the pebble level helping to relate them to the macroscopic response. There have been studies done on binary systems assuming the pebbles to be elastic and non-crushable by Annabattula et al. [2]. But in reality the pebbles are brittle in nature and prone to fail. Hence, it is necessary to understand the behavior of crushable assemblies for a better and

safer design. Some of the recent studies have considered the crushing behavior of mono-sized pebble assemblies by incorporating certain damage laws [3–5]. However, the produced pebbles vary in size. Hence, in this paper, we have considered a binary crushable assembly and studied the influence of initial PF, friction between pebbles ( $\mu$ ), size distribution and topology on the crushing behavior of the pebbles in the assembly, along with the macroscopic stress–strain response. This study helps in understanding the macroscopic response and damage in terms of microscopic factors paving way to model a unified prediction model for a crushable granular assembly.

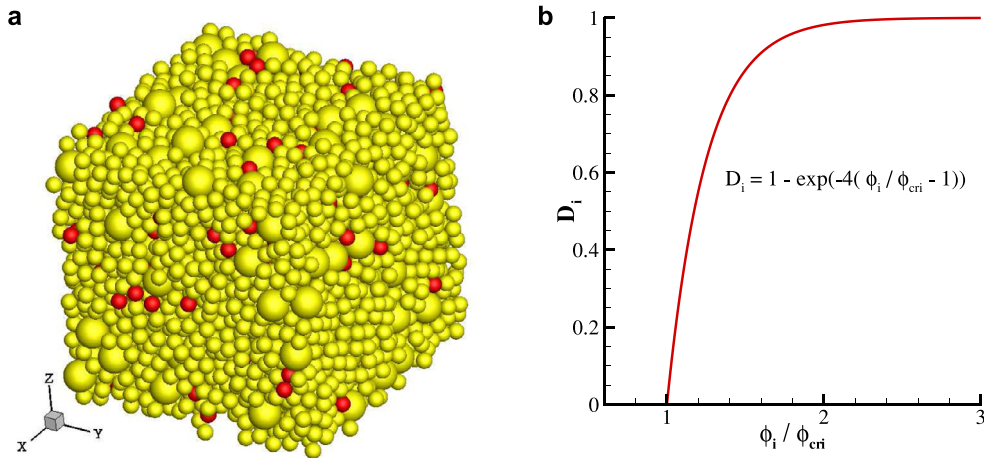
The outline of the papers is as follows. In Section 2, the simulation model and different parameters are discussed. In Section 3, the results concerning the influence of various factors are discussed for the crushable binary assembly followed by conclusions in Section 4.

## 2. Model

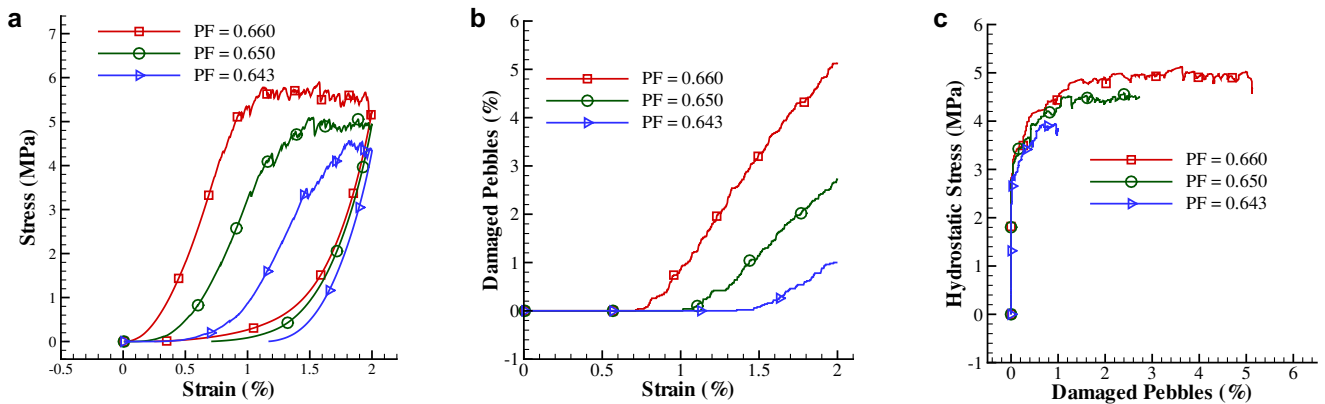
A periodic box consisting of binary crushable assembly of 5000 pebbles is taken as a representative volume element (RVE) (Fig. 1(a)). It is subjected to uniaxial compression in Z-direction up to a macroscopic strain of 2% and then unloaded to a

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**Fig. 1.** (a) Representative volume element of the pebble assembly showing the damaged pebbles (red). (b) Damage law used for the accumulation of damage of the pebble. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article).



**Fig. 2.** (a) Average stress–strain response and number of damaged pebbles as function of (b) strain and (c) hydrostatic stress-state of binary assemblies for various initial PFs. The simulations have been performed with  $N^* = 8$ ,  $r^* = 0.5$  and  $\mu = 0.1$ .

stress-free state. The pebbles are assumed to be spherical, with an elastic modulus of 90 GPa and Poisson's ratio of 0.25 (properties of OSi). Furthermore, a pebble is considered as failed during the simulations if the strain energy in the pebble reaches a critical failure energy obtained from experiments. Crush experiments on pebbles of various sizes show size dependence of critical failure energy [6,7]. The crush energy also varies over a range for a particular size [5]. However, we have assumed uniform failure energy (average value obtained from the experimental distribution) for pebbles of a given size in this paper for simplicity. Larger pebbles are assigned with higher crush energies compared to smaller pebbles in accordance with the experimental observations [7]. Damage of pebbles can be incorporated in the simulations by various damage laws [3,4]. A damage accumulation law, proposed by Annabattula et al. [3] is used for accounting the damage of the individual pebbles in this study. Damage can be considered as reduction in the load carrying capacity of a pebble and hence the damage is represented as the reduction of elastic modulus of the pebble. Damage starts when the strain energy of a pebbles reaches the failure crush energy (Fig. 1(b)). The stored elastic energy of the pebbles is calculated by the normal contact forces (only) exerted by the neighbouring contacting pebbles using Hertzian contact law. Simulations are carried out using an in-house DEM code (DEM\_KIT) [8]. The simulations are performed for different initial PF, friction coefficient between the pebbles and various binary pebble distributions varying size and relative number. The binary system used in this study is characterized by the following two non-dimensional parameters: number ratio ( $N^*$ ) and radius ratio ( $r^*$ )

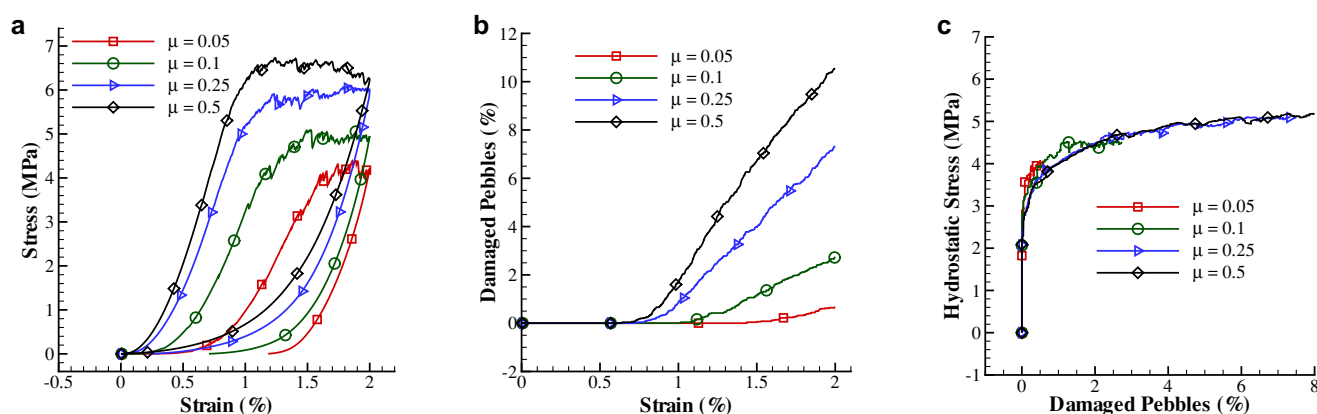
given by

$$N^* = \frac{N_s}{N_g}, \quad r^* = \frac{r_s}{r_g}, \quad (1)$$

where  $N_s$  is number of small pebbles,  $N_g$  is number of large pebbles,  $r_s$  is the radius of small pebble and  $r_g$  is the radius of the larger pebble. The macroscopic stress response and the pebble damage have been studied and compared with respect to number ratio  $N^*$  and radius ratio ( $r^*$ ). In order to understand the effect of initial configuration on the response of the system, three different configurations are created using a random closed packing algorithm. The responses from simulations are similar for a given case with 3 random realizations (results not shown). Hence in the following results are shown only for one random realization in all cases.

### 3. Results and discussion

In this section, we present the results of the DEM simulations for crushable binary assemblies. We have studied the effect of various factors on damage and macroscopic stress response. The normal stress developed in the direction of loading (Z-direction) is referred to as stress and average of the normal stress in X, Y and Z directions is termed as the hydrostatic stress. Hydrostatic stress is chosen as one of the parameters as it is the first invariant of the stress tensor. Fig. 2 shows the macroscopic stress and damage response of systems with different initial packing fractions (PF). The other system parameters ( $N^* = 8$ ,  $r^* = 0.5$  and  $\mu = 0.1$ ) are held



**Fig. 3.** (a) Average stress–strain response and number of damaged pebbles as a function of (b) strain and (c) hydrostatic stress-state of binary assemblies for various friction values. The simulations have been performed with  $N^* = 8$ ,  $r^* = 0.5$  and  $PF = 0.650$ .

constant for the simulations shown in Fig. 2. Fig. 2(a) shows the macroscopic stress–strain response for different initial PF. It can be observed that with increase in initial PF, the system exhibits increase in stiffness. Fig. 2(b) shows a decreasing trend in damage with decrease in initial PF. This is due to possible rearrangements in the case of loosely packed assemblies (lower PF), delaying the failure of pebbles with respect to strain. However, the evolution of the damage in the system as a function of hydrostatic stress as shown in Fig. 2(c) gives new insight into the damage mechanism. Fig. 2(c) shows that the failure is governed by the hydrostatic stress developed in the system. Even though we started with different initial PFs, the failure in the systems seems to be almost same when viewed with respect to the hydrostatic stress developed in the system. The damage in the system with respect to strain shows a strong influence of initial PF (Fig. 2(b)) whereas when seen with respect to hydrostatic stress the influence is greatly reduced. From Fig. 2, it can be observed that the uniaxial stresses developed in the system with varying initial PFs are plateauing at different values but the hydrostatic stress developed seems to be almost equal. The maximum stress developed in the loading direction is decreasing with decrease in the initial PF (Fig. 2(a)) due to rearrangement of pebbles facilitated by the loose packing. However, the hydrostatic stress developed is almost same as shown in Fig. 2(c). From these observations, the system seems to rearrange to reduce the deviatoric stresses developed due to loading.

Fig. 3 shows the effect of friction ( $\mu$ ) on the macroscopic responses for a given  $PF = 0.645$ ,  $N^* = 8$  and  $r^* = 0.5$ . The systems with higher friction coefficient are stiffer as observed from Fig. 3(a). The maximum stress value (at plateau formation) is decreasing with reduction of friction (Fig. 3(a)). For lower values of friction, due to ease of rearrangements, damage of the pebbles is delayed with respect to the strain developed in the system as seen in the Fig. 3(b). Fig. 3(c) shows the damaged pebbles as a function of hydrostatic stress developed in the system. As mentioned earlier, failure of pebbles seems to be uniquely governed by the hydrostatic stress irrespective of values of  $\mu$  as can be seen from Fig. 3(c). It shows that all curves are almost tracing the same path, indicating a strong correlation between failure and hydrostatic stress.

The influence of number ratio ( $N^*$ ) of pebbles in the binary assembly has also been studied. The radius ratio and friction coefficient are maintained constant as 0.5 and 0.1, respectively. The packing fraction is also required to be constant. However, due to the random closed packing algorithm used in generating assemblies with different  $N^*$ , there is slight variation in PF as shown in the legend of Fig. 4. Fig. 4(a) shows the effect of  $N^*$  on the macroscopic stress as function of applied strain. Fig. 4(c)

shows a decrease in the damage as the number of small pebbles are increased, i.e. as  $N^*$  increases. Even though we are replacing the larger pebbles which have higher crush energy compared to smaller pebbles, the damaged pebbles are observed to decrease for a given PF. The above observation may be attributed to two reasons. Firstly, smaller pebbles occupy the voids, giving more scope for rearrangement, making the system compliant. Secondly, apart from occupying the voids, they act as ball-bearings for the larger pebbles for rearrangements. However, this phenomenon is observed only to an optimal addition of smaller pebbles. Fig. 4(b) clearly shows that for the system with  $N^* = 4$  is compliant compared to  $N^* = 2$  supporting the phenomenon as discussed above. However, the system with  $N^* = 8$  shows higher stiffness. This counter intuitive response of increase in stiffness after certain value of  $N^*$  may be attributed to the departure from a binary like system to mono like system. This can be understood by noting that the system is moving from a mono to binary with increase in  $N^*$  and then by further increasing the number of smaller pebbles, it is again approaching a mono-like system but comprising smaller pebbles. These observations are made with respect to the strain developed in the system. Fig. 4(d) shows the damage in the system with respect to the hydrostatic stress developed. It shows that with addition of smaller pebbles which have low crush energies, damage is occurring at a slightly lower hydrostatic stress, which in turn should be the case as the average crush energy is decreased.

Fig. 5 shows the influence of radius ratio ( $r^*$ ) on macroscopic responses and percentage damage in the assembly as a function of applied strain. For  $r^* = 0.25$ , system shows less damage, less residual strain and higher stress, as seen in Fig. 5(a)&(b). The damage seems to increase as we increase the  $r^*$  value, even though we are replacing smaller pebbles which have lower failure energy with larger pebbles with higher failure energy. The counter intuitive behavior can be explained through Fig. 6. For low value of  $r^*$ , more number of larger pebbles are failing implying load is distributed mostly on larger pebbles. As the  $r^*$  is increasing the load is being shared among the both sizes and smaller pebbles having lower crush energy are failing more, as evident from Fig. 6(b).

The above observations show that damage in the granular systems is decreasing as the system becomes more compliant with respect to bed strain. Decrease in friction coefficient helps to reduce the damage. Also, there exists an optimal pebble distribution for a given packing fraction minimizing the damage. Furthermore, for a given size distribution (i.e., for a given  $N^*$  and  $r^*$ ), the damage can be well approximated as a unique function of hydrostatic stress irrespective of initial PF (see Fig. 2). Designing a compliant blanket design with optimal distribution helps to reduce the damage of the pebbles.

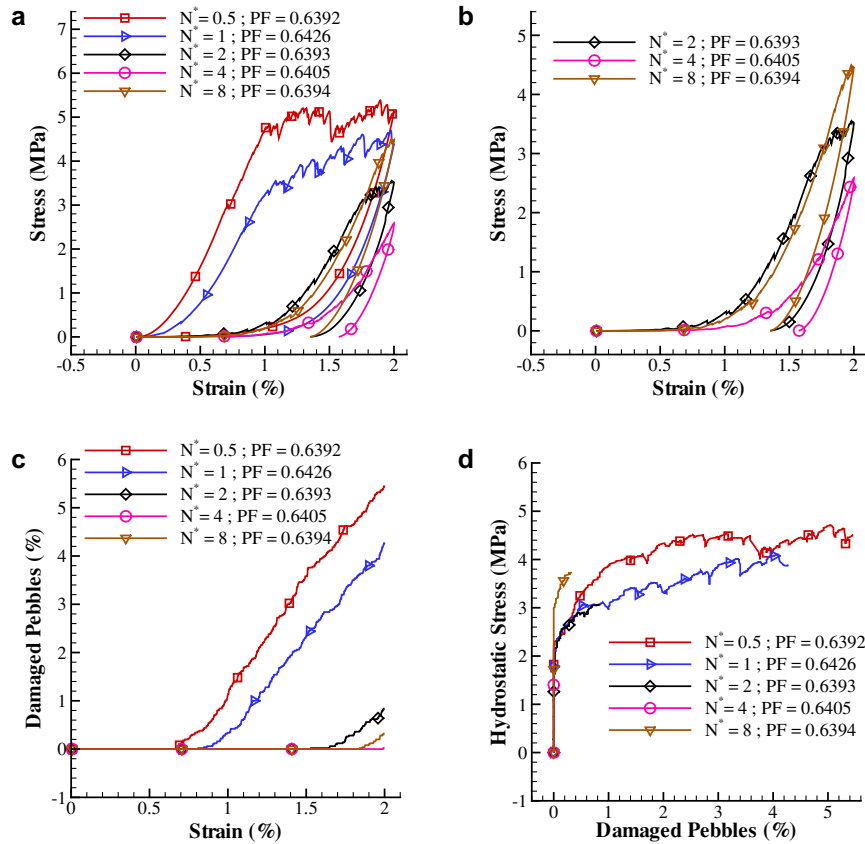


Fig. 4. (a)&(b) Average stress–strain response and number of damaged pebbles as a function of (c) strain and (d) hydrostatic stress-state of binary assemblies for various relative ratio of number of small to large pebbles. The simulations have been performed with  $\mu = 0.1$  and  $r^* = 0.5$ .

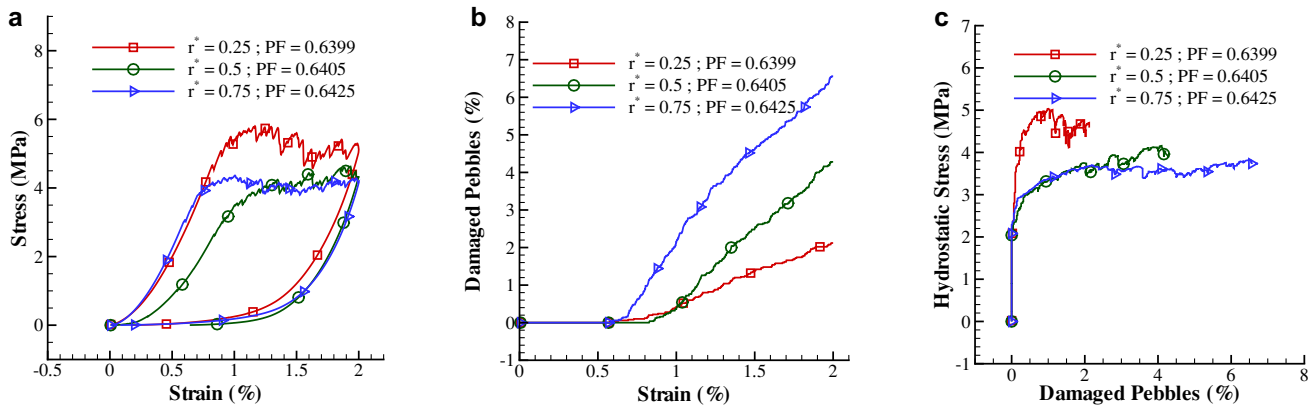
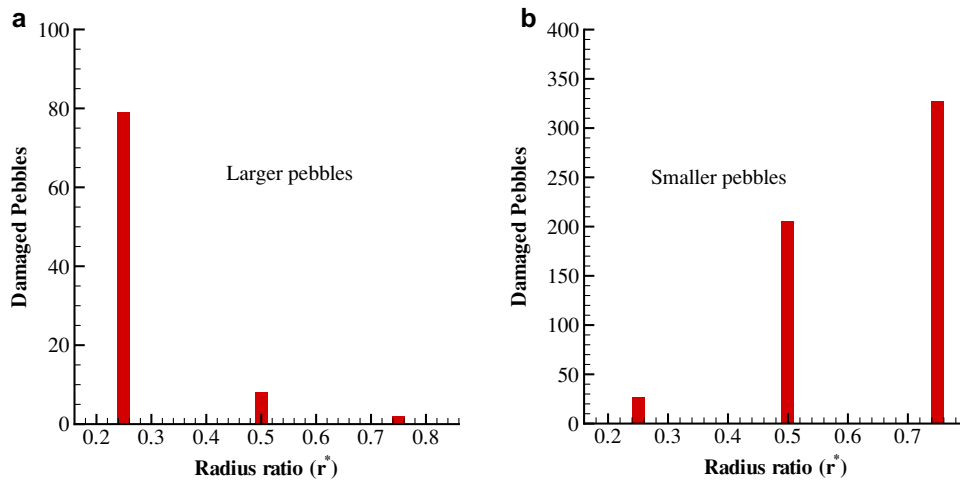


Fig. 5. (a) Average stress–strain response and number of damaged pebbles as a function of (b) strain and (c) hydrostatic stress-state of binary assemblies for various radius ratios. The simulations have been performed with  $\mu = 0.1$  and  $N^* = 1$ .

#### 4. Conclusions

Macroscopic stress–strain response of a binary crushable granular assembly has been studied and compared with respect to variation of PF, friction, relative radii and distribution of pebbles. Damage developed in the system has been studied with respect to the strain and also with respect to hydrostatic stress developed. The damage in the system is governed by the hydrostatic stress developed due to the loading. The damage shows a strong correlation with hydrostatic stress irrespective of initial PF for the system with similar pebble distribution. The rearrangement drives the system towards hydrostatic state of stress decreasing the deviatoric stress. The stress developed in the loading direction

(in case of uniaxial compression) is higher for the systems with less scope of rearrangement (viz. higher PF or high friction). The damage with respect to strain has been also investigated in order to estimate the damage behavior in terms of bed-strain and also for strain driven systems. The distribution and relative radii of pebbles also plays an important role in estimating the damage in the system with respect to both strain and hydrostatic stress. Despite the differences in initial PF and friction, values of the damage in the system shows a unique dependence on the hydrostatic stress developed for a given pebble size distribution (Figs. 2 and 3). Hence, it is more meaningful to describe damage of the system with respect to the hydrostatic stress state.



**Fig. 6.** Number of damaged pebbles of (a) larger size and (b) smaller size.

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