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# Crush probability analysis of ceramic breeder pebble beds under mechanical stresses

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ABSTRACT

A framework for analyzing crush events of individual ceramic pebbles in solid breeder blankets is developed by means of probabilistic methods. As a brittle material, ceramic breeder pebbles show considerable scatter in crush strengthen for single pebbles. The combination of the discrete element method and experimental data of crush loads provides the possibility of obtaining the overall crush probability of a pebble bed under compression. Furthermore, micro–macro relations are used to correlate the crush probability of pebbles with the overall stress level of the bed. Analysis of uniaxial compression of a mono-sized lithium-orthosilicate pebble bed is presented to demonstrate the application of this tool.

## 1. Introduction

Working as the tritium breeder material in Helium-Cooled Pebble Bed (HCPB) blankets in fusion reactors, lithium-based ceramic pebble beds are composed of nearly spherical shaped particles with diameter of approximately 0.5 mm [1,2]. Pebble beds are not only subject to severe conditions, e.g., thermo-mechanical loads and neutron irradiation, but due to their particular composition, also have complex behaviour. As a type of brittle materials, breeder pebbles may crush under thermo-mechanical loads inside fusion blankets. In mock-up experiments, severe crushing of pebbles has been observed after the out-of-pile tests (e.g., HELICA, [3,4]) and in-pile tests (e.g., EXOTIC-8, [5]). This phenomenon may affect the functionality of breeding blankets, e.g., the efficiency of tritium extraction and effective thermal conductivity of the beds, and it needs further investigation in a quantitative way to understand the crushing of pebbles under various conditions. In order to analyze the crush behaviour of individual particles inside the assembly and utilize crush load tests for single pebbles, we developed a probabilistic method for analyzing crush of individual pebbles under compressive stresses inside the blanket.

Crush load tests of single pebbles have been used for characterizing the strength of the material for decades [6–9]. Mean values of crush loads have been reported for different type of ceramic pebbles. For the probability distribution of the crush loads, a Weibull-type distribution has been suggested [10], while no fitting parameters have been reported. However, this simple test of pebbles has not yet been connected to the stress state, where individual pebbles are crushed inside the blankets. Ceramic pebbles are closely packed in a random state, and there are multiple contacts on the surface of a single pebble. The overall crushing of brittle granular material has been studied by different approaches, e.g., by a probabilistic method [11] and by a phenomenological approach [12]. For statistical analysis of the crushing events inside the assembly, the combination of the discrete element method (DEM) and experimental data provides the possibility of obtaining the crush probability of a granular medium in dependence on the mechanical stresses.

In this paper, we present the framework of a statistical analysis of crush inside ceramic breeder pebble beds in Section 2. Both the experiments and DEM simulation are discussed in Section 3. In Section 4, the overall crush probability of pebbles inside assemblies is calculated and some critical issues are discussed. Finally, some conclusions are drawn in Section 5.

## 2. Probabilistic method

The probability of a crushing event in an assembly will be given by the summation of the products of the probabilities of the following two independent events: (1) the probability that the strength of a particle has a value $\Phi$, as $p^S(\Phi)$ and (2) the probability that the inter-particle force is larger than $\Phi$. The first probability, $p^S$, is obtained by experimental investigations, $f_{\text{ave}}$ such as crush load tests of single particles, while the second one can be found

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in DEM simulations of relevant loading conditions. The inter-particle contact force distributions have been found to be independent of different loading levels if they are expressed by the normalized contact force as $P^f(f/f_{\rm ave})$ [13], where $f$ is the contact force and is the average contact force of all pebbles at current loading level. Therefore, we can write the following expression for the overall probability of a crushing event for each individual pebble,

$$P^O = \int_{F_{\min}}^{F_{\max}} p^S(\Phi)\cdot\left[\int_{\Phi/f_{\rm ave}}^{+\infty} p^f\left(\frac{f}{f_{\rm ave}}\right)\mathrm{d}\left(\frac{f}{f_{\rm ave}}\right)\right]\mathrm{d}\Phi \tag{1}$$

Differentiating the cumulative distribution function (CDF) $P^f(f/f_{\rm ave})$ gives the probability density function (PDF) of the force distribution $p^f(f/f_{\rm ave})$.

Since there are multiple contacts on any particle, there are two options for choosing the type of inter-particle contact forces which contributes to the crush, namely, (1) the maximum contact force on the particle and (2) all contact forces acting on the particle. For the first option, the assumption states that only the maximum contact force contributes to the crush of pebbles; while for the second option, the assumption is that the contributions of multiple contact forces depend on their magnitudes. The use of the first option is implemented directly by Eq. (1) by using the distribution of normalized maximum contact forces, $f^{(I)}_{\max}/f_{\rm ave}$,

$$P^O_{\max} = \int_{F_{\min}}^{F_{\max}} p^S(\Phi)\cdot[1 - P^f(\Phi/f_{\rm ave})]\mathrm{d}\Phi \tag{2}$$

Here $F_{\max}$ and $F_{\min}$ are the maximum and minimum strength observed in crush load experiments. The second option needs additional attention in interpretation of the experimental data and calculation of Eq. (1). In crush load tests on single pebbles, there are two contact forces acting simultaneously on the pebble from upper and lower plates. For simplification, we assume that the respective crush events introduced by these two contact forces are two mutually exclusive events and contribute equally to the overall crushing. Therefore, for each contact force, the crush probability is $p^S(F)/2$, where is the PDF of experimental data. After integrating Eq. (1), this provides the CDF of one contact force. For one particle, the multiplication of the coordination number $n_C$ is necessary to take into account all the contact forces acting on the surface. We have:

$$P^O_{\rm all} = \int_{F_{\min}}^{F_{\max}} p^S(\varPhi)\cdot\left[\int_{\varPhi/f_{\rm ave}}^{+\infty} \tilde{p}^f\left(\frac{f}{f_{\rm ave}}\right)\mathrm{d}\left(\frac{f}{f_{\rm ave}}\right)\right]\mathrm{d}\varPhi \tag{3}$$

where

$$\tilde{p}^f(f/f_{\rm ave}) = \sum_i i\cdot p^f(f|n_C = i)p(n_C = i)$$

Here $p(n_C = i)$ is the probability that a particle has $i$ contacting neighbors, the lower and upper limits of $i$ in mono-sized packing are 2 and 12, respectively.

During loading and unloading of the bed, the value of $f_{\rm ave}$ differs from one step to another. The average contact force represents the macroscopic stress state of the assembly. In order to correlate the crush probability of pebbles with the overall stress level of the bed, $f_{\rm ave}$ has been determined as a function of macroscopic hydrostatic pressure $\sigma_h$. According to the derivation in the previous work [13], this relation can be written as:

$$\sigma_h = \frac{\bar{n}_C\eta}{4\pi r^2}f_{\rm ave}. \tag{4}$$

Here $r$ is the radius of the particle, $\bar{n}_C$ indicates the average coordination number in the assembly and $\eta$ is the packing factor, i.e., the relative density, of the pebble bed. The packing factor ranges from 63% to 64% depending on the initial packing procedure [14]. After we solve Eq. (1), we have the overall crush probability as a function of the average contact force, as $P^O(f_{\rm ave})$. Considering Eq. (4), we link the overall crush probability to the macroscopic stress state as $P^O(\sigma_h)$. This gives the possibility of evaluating the probability of crushing inside the assembly with respect to the mechanical stress state.

## 3. Analysis of crushing events

In this section, the experimental data $p^S(F)$ and inter-particle force distribution $p^f(f/f_{\rm ave})$, which are the two essential parts to calculate the overall crush probability $P^O(f_{\rm ave})$ in Eq. (1), are discussed.

### *3.1. Crush load tests*

The crush behaviour of individual pebbles has been investigated in experiments. In this investigation, two types of $Li_4SiO_4$ pebbles, namely as-fabricated (OSI-07/1) and annealed (OSI-07/1-c) pebbles [2] have been tested. The pebbles have nearly spherical shapes with diameter ranging from 0.50 to 0.56 mm. After drying at 300 °C, the pebbles are loaded until crushed by two parallel glass plates in a glove box, and the crush loads are recorded. Details of the measurements are in Table 1, which includes the results from 140 samples of OSI-07/1 pebbles and 200 samples of OSI-07/1-c pebbles. Comparing to the as-fabricated pebbles, a 10% lower mean value with a relatively small standard deviation has been found for conditioned pebbles. This is mainly due to microstructural changes during an additional annealing process. The following Weibull distribution gives the CDF of the crush test data,

$$P^S(F) = \int_0^F p^S(\Phi)\mathrm{d}\Phi = 1 - \exp\left[-\left(\frac{F - F_0}{k}\right)^m\right], \tag{5}$$

where $F$ (N) represents the crush load measured in experiments, $m$ and $k$ are shape and scale parameters in the Weibull distribution, respectively, $f_0$ is the location parameter, and $F_0 = F_{\min}P^S(F)$ is the CDF of the strength of individual pebbles. A probabilistic analysis of the data using Bayes factors [15] showed a weak preference (odds of 60:40) in favour of a Gaussian distribution compared to a Weibull distribution. This is in agreement with the visual impression given by plotting the data in Weibull and Gaussian probability plots. In this work, we consider the Weibull function as the first attempt, and set. In Table 1, the parameters have been calculated by the maximum likelihood method based on the measurements. The fitting curves and the experimental data for these two types of pebbles under consideration are shown as PDF in Fig. 1.

**Table 1**
Parameters of the crush load tests (unit: N).

| Types | $F_{\rm mean}$ ± s.d. | $F_{\min}$ | $F_{\max}$ | $m$ | $k$ |
|---|---|---|---|---|---|
| 07/1 | 6.33 ± 1.81 | 3.68 | 13.04 | 1.29 | 2.82 |
| 07/1-c | 5.88 ± 1.14 | 3.12 | 8.56 | 2.16 | 3.02 |

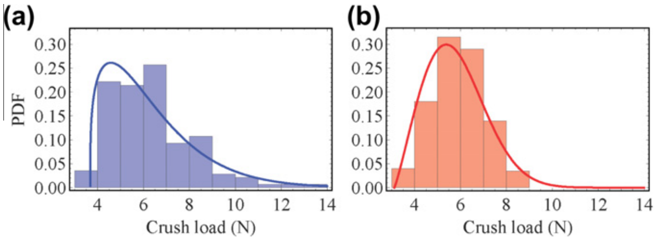

**Fig. 1.** The probability density of experimental data for $Li_4SiO_4$ pebbles: (a) OSI 07/1 and (b) OSI 07/1-c. The best fit curves are shown as solid lines for comparison.

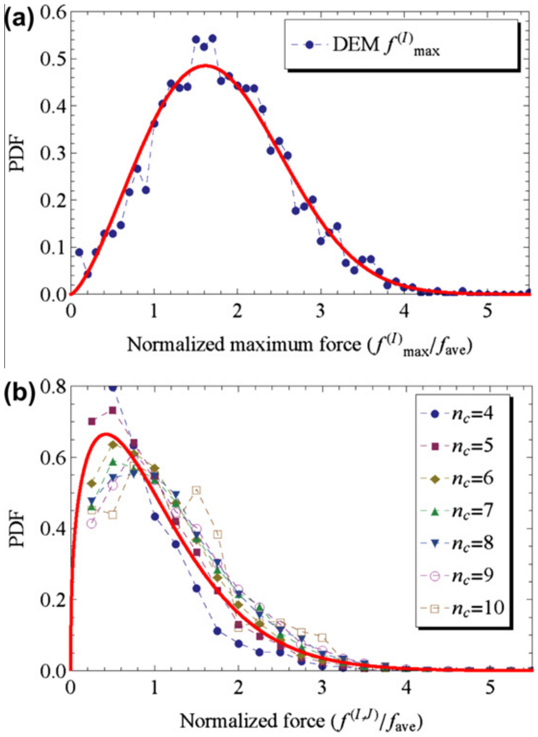

**Fig. 2.** DEM simulation of uniaxial compression: probability density functions of (a) normalized maximum contact forces and (b) all contact forces.

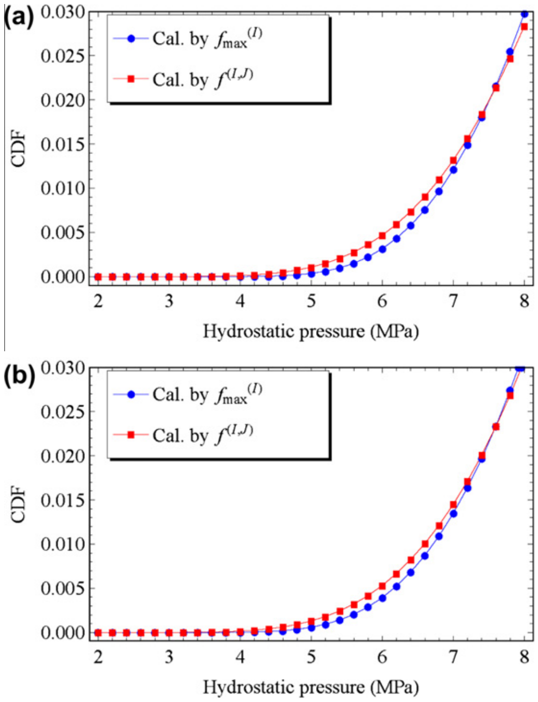

**Fig. 3.** Cumulative distribution functions for crushing of individual pebbles inside the bed: (a) as-fabricated pebbles; (b) conditioned pebbles, as functions of macroscopic stress state, calculated by (1) maximum contact forces in Eq. (2) all inter-particle contact forces in Eq. (7).

### 3.2. Inter-particle contact forces

An inhomogeneous stress state inside HCPB blankets can occur due to complicated structural and thermal boundary conditions. As an application, analysis of uniaxial compression of a mono-sized $Li_4SiO_4$ pebble bed is presented here to demonstrate this approach. From DEM simulations [13], probability distributions of normalized contact forces have been found to be independent of the stress level, i.e., the contact forces scale with the level of the applied stresses, and the average coordination number $\bar{n}_C$ is explicitly expressed by the stress state. The PDF of the normalized contact forces can be expressed well by a Weibull distribution with two parameters, $m$ and $k$. Here, $f^{(I)}_{\max}/f_{\text{ave}}$ represents the normalized maximum contact force on particle $I$, and shows the normalized inter-particle contact force between particles $I$ and $J$. The fitting curves and simulation data are shown in Fig. 2: (a) PDF of normalized maximum forces (with fitting parameters of $m = 2.40$ and $k = 2.02$), and (b) PDF of all contact forces ($m = 1.37$ and $k = 1.10$). In Fig. 2b, distributions of simulation data are shown with different coordination number and the fitting parameters are extracted from the overall force distribution. The cases with different number of contacts have similar force distributions, suggesting $p^f(f|n_C = i) = p^f(f)$, and we have.

$$\sum_i i \cdot p^f(f|n_C = i)p(n_C = i) = \bar{n}_C \cdot p^f(f). \tag{6}$$

Therefore, Eq. (3) can be written as

$$P^O_{\text{all}} = \frac{\bar{n}_C}{2}\int_{F_{\min}}^{F_{\max}} p^S(\Phi)\cdot[1 - P^f(\Phi/f_{\text{ave}})]\mathrm{d}\Phi \tag{7}$$

Comparing to Eq. (2), the force distribution $P^f$ is different in Eq. (7), which represents the CDF of all contact forces in the assembly, and the coefficient $\bar{n}_C/2$ introduces the contributions of all neighbor contacts.

## 4. Overall crush probability inside beds

Integrating Eqs. (2) and (7) gives the final results shown in Fig. 3 as a function of overall hydrostatic stress level. There are two calculations, based on different assumptions for the crush mechanism, namely, maximum contact force dominated and considering all contact forces, shown in each sub-figure, respectively. Fig. 3a and b show the predictions for different types of pebbles, i.e., as-fabricated and conditioned pebbles, respectively. Though a 10% lower mean value has been found for conditioned pebbles, the overall crush probability shows only small variation for these two types of pebbles. This suggests that the mean value of crush loads is not sufficient for characterizing the strength of pebbles, whereas in literature usually only the mean values have been reported. Therefore, the distribution of crush loads provides more information and a statistical approach will be essential for presenting the strength of ceramic pebbles. These predictions are calculated up to 8 MPa since this is a common maximum hydrostatic stress state inside $Li_4SiO_4$ ceramic region [4], and the predicted crush probability at this stress level is beyond the onset of crushing.

The overall crush probability $P^O(\sigma_h)$ presented here is a value used to characterize the level of crushing inside an assembly. This quantity will be important for the design and diagnostics to avoid massively crushed pebbles inside the breeder zone during operation, which may affect the functionality of fusion blankets. There are several possibilities to reduced the fraction of crushed pebbles: (1) using material with increased crush strength and also high crush resistance under thermal cycle and neutron irradiation, (2) design of structures to reduce thermal stresses by an active heat removal system, (3) using loose packing to reduce the resulting stress level. However, for a loose packed bed, the reduction of effective thermal conductivity, reduction of overall tritium breeding ratio and formation of gaps between pebbles and walls will be issues that may limit this solution.

A specific CDF value, e.g., 1%, states the average crush probability of one individual pebble under a certain macroscopic stress level. For large assemblies, if we define the onset of crushing is, for instance, $P^O = 1\%$, further crush events may occur after the onset of crushing. The progressive crush analysis could be performed by DEM simulation considering the previous crush events. Furthermore, this work can be extended to study the crush of pebbles under other loads in fusion blanket applications, such as thermal stresses and neutron irradiation. The degradation of pebbles under neutron flux could be taken into account by replacing Eq. (5) with the corresponding crush load test data of irradiated pebbles.

## 5. Conclusion

In this work, a probabilistic method for ceramic pebble beds used in fusion blankets has been proposed to quantitatively analyze the crush events of individual pebbles. The combination of crush load tests of single pebbles and DEM simulations provides the possibility of achieving the macroscopic prediction based on simple experiments and multi-scale modelling. Statistical analysis of the crush load tests is suggested to replace the mean value of crush loads to better understand the crush behaviour of pebbles. Finally, the crush events of individual pebbles are expressed as a function of the macroscopic stress state, which can be used for design and optimization of HCPB blankets.

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## References

[1] J. Reimann, L. Boccaccini, M. Enoeda, A.Y. Ying, Fusion Eng. Des. 61–62 (2002) 319–331.
[2] R. Knitter, B. Alm, G. Roth, J. Nucl. Mater. 367–370 (Part 2) (2007) 1387–1392.
[3] G. Dell'Orco, A. Ancona, A. DiMaio, M. Simoncini, G. Vella, J. Nucl. Mater. 329–333 (Part 2) (2004) 1305–1308.
[4] Y. Gan, M. Kamlah, J. Nucl. Mater. 386–388 (2009) 1060–1064.
[5] G. Piazza, A. Erbe, R. Rolli, O. Romer, J. Nucl. Mater. 329–333 (2004) 1260–1265.
[6] G. Schumacher, M. Dalle Donne, S. Dorner, J. Nucl. Mater. 155 (1988) 451–454.
[7] G. Piazza, J. Reimann, E. Gunther, R. Knitter, N. Roux, J.D. Lulewicz, Fusion Eng. Des. 58–59 (2001) 653–659.
[8] K. Tsuchiya, H. Kawamura, S. Tanaka, Fusion Eng. Des. 81 (8–14) (2006) 1065–1069.
[9] N. Zaccari, D. Aquaro, J. Nucl. Mater. 386 (2009) 1078–1082.
[10] P. Gierszewski, Fusion Eng. Des. 39–40 (1998) 739–743.
[11] G. Marketos, M.D. Bolton, J. Mech. Phys. Solids 55 (10) (2007) 2142–2156.
[12] I. Einav, J. Mech. Phys. Solids 55 (6) (2007) 1298–1320.
[13] Y. Gan, M. Kamlah, J. Mech. Phys. Solids 58 (2) (2010) 129–144.
[14] Y. Gan. Thermo-Mechanics of Pebble Beds in Fusion Blankets, PhD Thesis, Universität Karlsruhe, FZKA 7455, 2008.
[15] H. Riesch-Oppermann, O. Kraft, in: Proceedings of the 12th International Conference on Fracture, Ottawa, 2009.